Exploring the limits of the thermodynamic model of heavy-ion collisions with respect to particle ratios

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Many models have been used to predict the shape of particle energy spectra and also to attempt to understand the underlying physics that dictates this shape. The thermodynamic model of heavy-ion collisions as described by das Gupta and Mekjian in Reference [1] predicts the shape of the energy distribution of composite nuclei based on that of protons. To do so they suggest that

$$\frac{d^3N_A}{d^3p_A}(p_A) = \exp\left(\frac{\mu_A}{T}\right) g_A \frac{V}{h^3} \exp\left(-\frac{\sqrt{m_A^2c^4+p^2c^2}}{T}\right) \qquad (1)$$

where $\mu_A = N\mu_n + Z\mu_p$ for a composite particle A that has N neutrons and Z protons. Here, p_A is the momentum per nucleon of the composite particle, g_A is the spin degeneracy factor of the composite particle, $\mu_{n,p}$ are the chemical potentials for neutrons and protons respectively, V is the volume, and T is the temperature. We would like to investigate the difference in the shape of energy spectra for a given isotope from two different sources under similar conditions. Using Equation (1) we can take the ratio of the energy spectra for a chosen isotope from one reaction system to the energy spectra for the same isotope from a different reaction system. By doing so, we can compare our experimentally determined ratios to the change in chemical potential between the two systems. The reaction systems were chosen to have the same Z (to avoid significant Coulomb differences) and different asymmetries where the asymmetry is defined as $\delta = \frac{N-Z}{A}$. We will refer to the ratio of these energy spectra moving forward as the Isotope Particle Ratio (IPR).

When we investigate the IPR for large systems and at a specific kinetic energy we note that most of the terms cancel out and we are left with

$$\frac{\frac{dN_B}{dE}}{\frac{dN_A}{dE}} = \exp(\Delta\mu/T) \qquad (2)$$

where B and A represent the spectra (as a function of center of mass kinetic energy instead of momentum) from two different reaction systems. The resulting $\Delta\mu = N\Delta\mu_n + Z\Delta\mu_p$ implies that the value of this ratio depends only on the neutron and proton content of the emitted isotope as well as the difference in the chemical potentials for neutrons and protons between these two systems.

One of the primary uses of isotope ratios has been isoscaling which is typically performed using integrated particle yields as originally described in Ref. [2]. In most experiments the inability to achieve a large amount of statistics for heavy particles requires the use of integrated yields instead of investigating

the ratio of the spectra. In the thesis experiment performed by Z. Kohley and described in Ref. [3], measurements using the 4π NIMROD array were taken measuring the reaction products from both ^{64}Zn+^{64}Zn and ^{70}Zn+^{70}Zn (system *A* and B respectively in Eq. (2)) at 35 MeV per nucleon. In this experiment, kinetic energy spectra were measured for a broad range of particle types with isotopic resolution up to *Z=17* in many detectors with *Z=20* measured in a select few.

It is important to note that Eq. (1) is intended to use the true differential multiplicity in the calculation and not the measured one. Any losses due to detection efficiency can significantly change the results of the composite spectra. Unfortunately, completing a full efficiency corrected set of data for each isotope would be prohibitive and so to approximate this effect we include only detectors that behaved similarly in both sets of data. This removes the bulk of any differential efficiency that would occur between the two reaction systems. We then construct the kinetic energy spectra per event whereupon taking the ratios of the spectra would cause any efficiency corrections to cancel out to first order.

The primary goal of this analysis is to investigate large composite particles and the different trends these ratios provide. We begin by investigating the isotopes as displayed in Fig. 1 so that each panel represents a different value of *N-Z*. All particles of the given isotope, regardless of detection

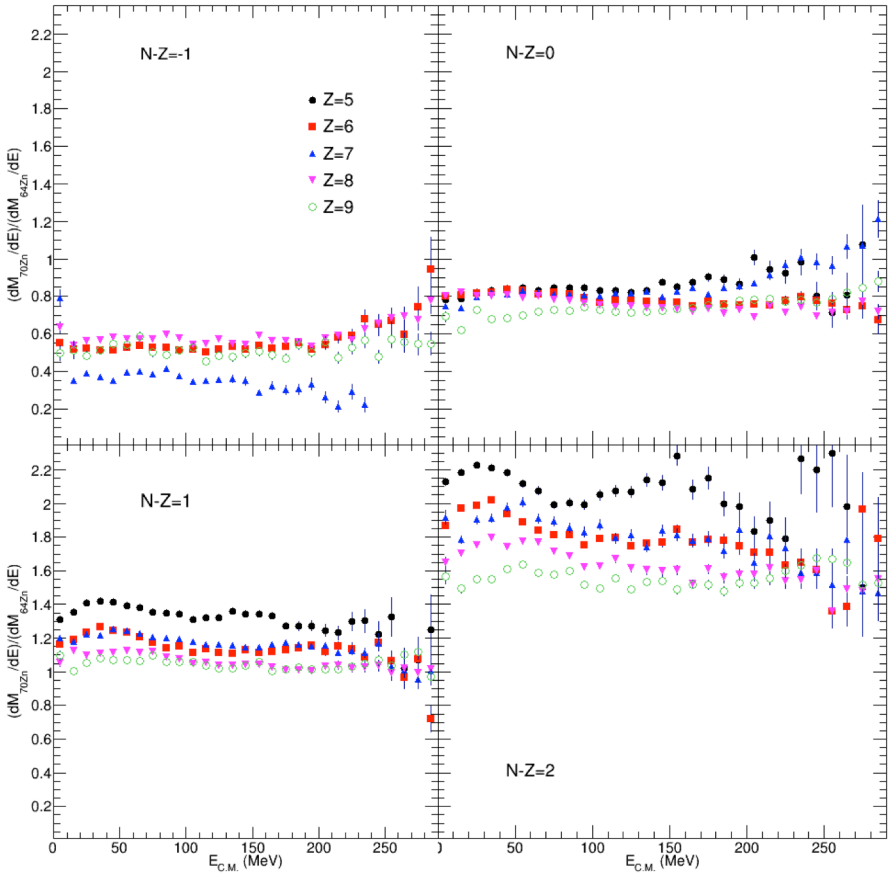

FIG. 1. The IPR for particles with Z=5 through Z=9. Each panel represents the isotopes that fall into the category N-Z = -1 (upper left), 0 (upper right), 1 (lower left) and 2 (lower right). Each plot shows the IPR as a function of center of mass kinetic energy

location or energy, are included in the figure with statistical uncertainties provided as error bars. In the figure, only particles with $4 \leq Z \leq 9$ are provided for demonstration. We omit particles with $Z \leq 4$ due to the expectation that secondary decay is causing an added effect.

Several interesting results of this figure immediately present themselves. As a function of center of mass kinetic energy, the ratio of the spectra for a given fragment is remarkably flat. Assuming that the thermodynamic model is valid, one can understand this by realizing that the average temperature between the two systems is approximately equal. This implies that the ratio in Equation (2) is energy independent.

In addition, as the neutron excess of the isotope increases the value of the ratio also increases. Each class of isotopes (where a class is given by a constant *N*-*Z* value) rises together in a consistent manner. This implies that $\Delta\mu_n > 0$, while the value of the ratio of the symmetric isotopes (and proton-like isotopes) suggests $\Delta\mu_p < 0$, and that the magnitude of $\Delta\mu_p$ is greater than that of $\Delta\mu_n$. This result helps to explain an initially unexpected trend primarily visible in the *N*-*Z*=1 and 2 panels; as *Z* increases the value of the IPR decreases. This can be explained mathematically based on the results of the sign and magnitude of the chemical potentials. Let *X* be an isotope with *N* neutrons and *Z* protons, with isotope *Y* having *N*+1 neutrons and *Z*+1 protons so that the value of *N*-*Z* remains constant. Simple algebra will show $\Delta\mu_Y < \Delta\mu_X$ implying that the IPR for particle *Y* would be less than particle *X*. With a different choice of the two systems this trend may be reversed as the change in chemical potential for protons and neutrons may change, however this trend for the larger values of *N*-*Z* is in agreement with the results from the symmetric data.

As mentioned previously, in each panel we see agreement between the different isotopes that have the same value of *N*-*Z*. This holds quite well with the lone exception of the proton-like ^{13}N. At this time, it is uncertain why this particular isotope behaves in such a way. In order to investigate this trend more, a horizontal line was fit to each isotope and the result was plotted as a function of *N*-*Z*. In Fig. 2 we can see the fit results for four different groups of *Z* values over the full range of measured isotopes for that given *Z*. It appears that within each group of isotopes up through *Z=14* the fits follow the same general pattern of an exponential increase with the value of *N*-*Z* in reasonable agreement with its nearest neighbors. As expected, each group decreases its value relative to the previous set of lower *Z* values. This trend is seen for all types of isotopes regardless of their *N*-*Z* value.

Once the measurements reach approximately *Z=15*, the trend begins to change resulting in the final result presented here. This value of *Z* represents the value where half of the charge of the beam has been measured in one particle so it is reasonable to assume that the mechanisms that create these isotopes are no longer governed by the chemical potential in the same way that the lighter isotopes are. While *Z=15* and *16* seem to follow the same general trend that the lighter isotopes do, their values and separation are significantly different. In the *Z=17-20* region, the pattern changes significantly not only in values of the fits, but also shape. This implies that the limit where the mechanics of the thermodynamic model break down is around half of the charge or mass of the beam. The most reasonable physical explanation is that the large particles most likely represent residues of the emitting source instead of the particles being emitted from the source.

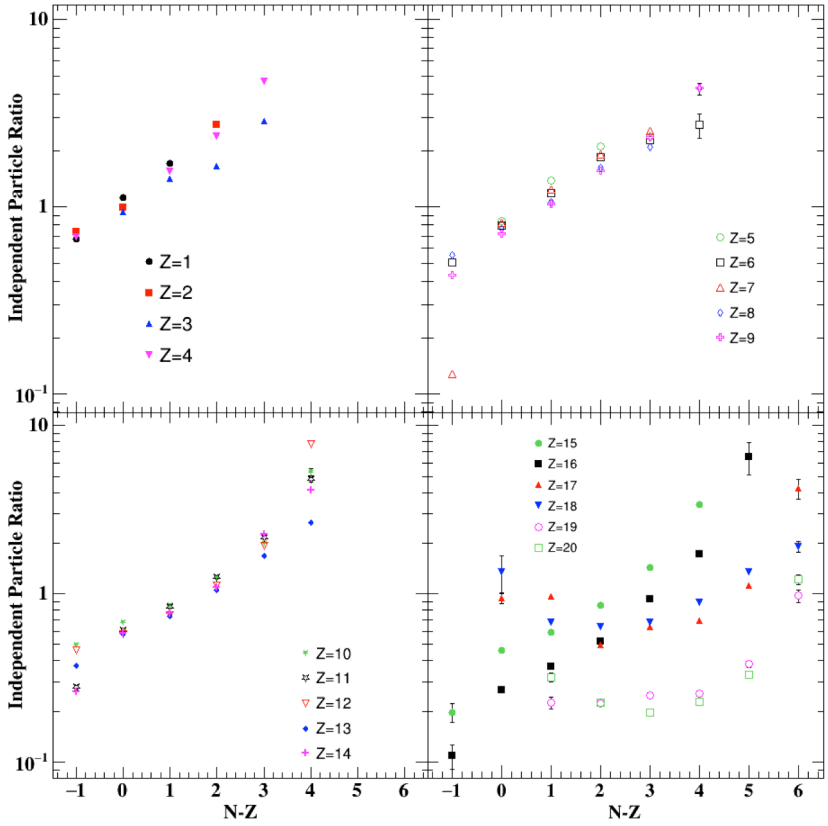

FIG. 2. The fit value for a horizontal line for each isotope from the ratio of spectra. The data is split into four groups where the first three groups (top row and lower left) show agreement within a panel while the lower right panel ($15 \leq Z \leq 20$) shows what appears to be a limit where this agreement breaks down.

[1] S. Das Gupta and A.Z. Mekjian, Phys. Rep. **72**, 131 (1981).

[2] M.B. Tsang *et al.*, Phys. Rev. Lett. **86**, 5023 (2001).

[3] Z. Kohley, Ph.D. Thesis, Texas A&M University, 2010.